UDC 62-50

DIFFERENTIAL GAMES OF ENCOUNTER FOR DISTRIBUTED-PARAMETER SYSTEMS

PMM Vol. 39, № 5, 1975, pp. 772-779

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(Received February 2, 1975)

We determine sufficient conditions for the successful completion of an encounter process for a controlled system of specified purpose, described by a differential equation in Hilbert space with, generally, unbounded operator. The arguments are based on a method of constructing the resolving controls on the feedback principle playing here the role of Krasovskii's extremal aiming principle /1, 2/. The class of controlled systems to be examined includes, for example, certain systems described by parabolic and hyperbolic partial differential equations /3—11/. The paper abuts the investigations in /1, 2, 12—17/.

1. We consider the controlled system

$$x^{\cdot}(t) = A_1 x(t) + A_2 u - A_3 v + w(t), \quad t_0 \leqslant t \leqslant \vartheta \tag{1.1}$$

Here $x(t)$ is the plant's state at instant t, being an element of a real Hilbert space H_1; u, v are control parameters subject to the constraints $u \in P \subset H_2$, $v \in Q \subset H_3$; P, Q are convex bounded closed sets, H_2, H_3 are real reflexive Banach spaces; A_i $(i = 1, 2, 3)$ is a linear operator from H_i into H_1, bounded for $i = 2, 3$; A_1 generates in H_1 a strongly continuous semigroup /18/ $\{F(t), t \geqslant 0\}$ satisfying the condition $\|F(t)\|_1 \leqslant \exp \omega t$, $t \geqslant 0$ for some ω ($\|\cdot\|_i$ is the symbol for the norm in H_i); $w(t)$ is a given function, integrable on $[t_0, \vartheta]$ (here and subsequently, integrability is understood in Bochner's sense, (strong) measurability is understood in the Lebesgue sense, the derivative in (1.1) is understood as the limit with respect to the norm in H_1 of the corresponding finite-difference relation /18/).

The encounter problem to be examined is the following. A closed set M is given in space H_1. We are required to find a method for forming the control u by the feedback principle ($u[t] = u(t, x(t))$), ensuring the contact of point $x(t)$ with target M at the instant $t = \vartheta$ when control v is formed by any method developing a measurable realization $v[t]$ with values in Q for almost all t. (In particular, we do not exclude the method of forming control v, which uses at each instant t information on the control $u[t]$ at this same instant).

The encounter problem described was analyzed in /13/ wherein, in particular, a mathematical formalization of it was proposed, the necessary and sufficient solvability conditions were stated, and a method for constructing the resolving controls was given (*). In the general case the verification of these conditions and the construction of the encounter strategies are very difficult. However, as in /1, 2, 15/, we can find (and this is the aim of the present paper) a wide class of controlled systems when such a verification can be effected by a sufficiently simple and effective concretization of the encounter strategy construction procedure found in /13/.

2. Let us make the problem statement more precise. A rule which associates a set $U(p) \subset P$ with each position $p = \{t, x\}$, $t \in [t_0, \vartheta]$, $x \in H_1$, is called a strategy U. Let Δ be a finite partitioning of $[t_0, \vartheta]$ by the points τ_i $(\tau_0 = t_0$, $\tau_i < \tau_{i+1}$, $i = 0, \ldots, m(\Delta))$, $\delta(\Delta) = \max_i(\tau_{i+1} - \tau_i)$. The function $x(t)_\Delta = x(t, p_0, U)_\Delta$, $t_0 \leqslant t \leqslant \vartheta$, equal to

$$x(t)_\Delta = F(t - t_0)x_0 + \int_{t_0}^{t} F(t - \xi)(A_2 u[\xi] - A_3 v[\xi] + w(\xi))\,d\xi$$

is called the motion of system (1.1) from the position $p_0 = \{t_0, x_0\}$, corresponding to strategy U. Here $u[t]$ is the control dictated by strategy U: $u[t] = u[\tau_i] \in U(\tau_i, x(\tau_i)_\Delta)$, $\tau_i \leqslant t < \tau_{i+1}$; $i = 0, \ldots, m(\Delta)$; $v[t]$ is some measurable realization of control v for almost all t with values in Q.

The original control problem is formalized in the following way /13/. Let M^ε be a closed ε-neighborhood of M in H_1.

P r o b l e m 2.1. Construct the strategy U with the property: for every number $\varepsilon > 0$ there exists a number $\delta_0 > 0$ such that the inclusion $x(\vartheta)_\Delta \in M^\varepsilon$ is fulfilled for all motions $x(t)_\Delta = x(t, p_0, U)_\Delta$ with $\delta(\Delta) \leqslant \delta_0$.

Sufficient conditions for the solvability of Problem 2.1 are found in Sect. 3.

N o t e 2.1. The concept of motion $x(t)_\Delta$ corresponds to the concept of the solution of Eq. (1.1) in the generalized sense (see /3–7/). As a matter of fact, if the initial state x_0 belongs to the domain of operator A_1 and the realization of control v (and the perturbation w) is a sufficiently smooth time function, then the motion of each interval $[\tau_i, \tau_{i+1})$ is the classical solution of Eq. (1.1).

3. *For what follows we require certain notions from /13/. Let there be a system of* nonempty sets $B_t \subset H_1$, $t_0 \leqslant t \leqslant \vartheta$. For $x \in H_1$ we denote $r(t, x) = \inf \| x - y \|_1$, $y \in B_t$. Let $Z(x, B_t)$ be the collection of all weak limits of all possible sequences $\{y_k\} \subset B_t$, weakly convergent in H_1, minimizing the functional $\| x - y \|_1$. Obviously, $Z(x, B_t) \neq \phi$. The strategy

$$U^e = U^e(t, x) = \{u^e \mid \langle A_2^*(y - x), u^e \rangle_2 = \max_{u \in P} \langle A_2^*(y - x), u \rangle_2,\ y \in Z(x, B_t)\} \tag{3.1}$$

is said to be extremal to the sets B_t, $t_0 \leqslant t \leqslant \vartheta$ /13/. (Here and later on, A^* is the operator adjoint to operator A, $\langle f, z \rangle_i$ is the value of functional f on the element $z \in H_i$). The concept of strategy U^e given here is a natural development of the con-

(*) Analogous questions are discussed in: Osipov, Iu. S., Differential games in distributed-parameter systems. Reports Abstracts Third All-Union Conf. on Game Theory. Odessa, 1974.

cept of an extremal strategy from /1, 2, 15, 16/.

Note 3.1. By definition the strategy U is, generally speaking, a multivalued mapping of $[t_0, \vartheta] \times H_1$ into P. It is not difficult to see that everything said in the present paper is preserved if we restrict ourselves to the single-valued mappings. Here, by U^e we should understand a rule associating a certain element u^e, defined in (3.1), with each position p.

The functions $u(t)$ $(v(t))$, measurable on some time interval, for almost all t with values in P (Q), are called programs. The system of nonempty sets B_t, $t_0 \leqslant t \leqslant \vartheta$ is said to be strongly u-stable if the following condition is fulfilled: for every quantities $t_* \in [t_0, \vartheta)$, $t^* \in (t_*, \vartheta]$, $x_* \in B_{t_*}$ and the program $v(t)$, $t_* \leqslant t \leqslant t^*$, there exists a program $u(t)$, $t_* \leqslant t \leqslant t^*$, with the property

$$F(t^* - t_*)x_* + \int_{t_*}^{t^*} F(t^* - \xi)(A_2 u(\xi) - A_3 v(\xi) + w(\xi))\,d\xi \in B_{t^*}$$

The following statement, revealing a property of strategy U^e, is valid.

Lemma 3.1. The strategy U^e extremal to the system of strongly u-stable sets B_t, $t_0 \leqslant t \leqslant \vartheta$, possesses the property : for every number $\varepsilon > 0$ there exist numbers $\delta_0 > 0$ and $\alpha > 0$ such that for any motion $x(t)_\Delta = x(t, \{t_0, x_0\}, U^e)_\Delta$ the inequality $r(t, x(t)_\Delta) < \varepsilon$, $t_0 \leqslant t \leqslant \vartheta$, is fulfilled if only $\delta(\Delta) \leqslant \delta_0$ and $r(t_0, x_0) \leqslant \alpha$.

The lemma is proved by the plan of proof of the analogous statements in /1, 14, 16/.

Under the condition of stability of sets B_t and in the presence of the bound $\|F(t)\|_1 \leqslant \exp \omega t$ the method of choosing controls $u^e[t]$ which generate motions $x(t, p_0, U^e)_\Delta$ in correspondence with rule (3.1), guarantees the inequality

$$r^2(\tau_{i+1}, x(\tau_{i+1})_\Delta) \leqslant (1 + \omega\delta(\Delta))\, r^2(\tau_i, x(\tau_i)_\Delta) + o(\delta(\Delta))$$

where $o(\delta)/\delta \to 0$ as $\delta \to 0$. Hence we conclude that for every number $\beta > 0$, all motions $x(t)_\Delta = x(t, p_0, U^e)_\Delta$ satisfy the inequality

$$r(t, x(t)_\Delta) \leqslant \beta \exp 2\omega(t - t_0), \quad t_0 \leqslant t \leqslant \vartheta$$

if only $\delta(\Delta) \leqslant \delta_0$, $r(t_0, x_0) \leqslant \alpha$, where $\delta_0 > 0$ and $\alpha > 0$ are sufficiently small numbers. This proves the lemma's assertion.

From Lemma 3.1 follows, trivially,

Theorem 3.1. Let there be a system of strongly u-stable sets B_t, $t_0 \leqslant t \leqslant \vartheta$, where $B_\vartheta \subset M$. If $r(t_0, x_0) = 0$, then strategy U^e, extremal to this system of sets, solves Problem 2.1.

In connection with Theorem 3.1 the question arises of the existence and the construction of a system of sets with the properties indicated. In such a system of sets we can choose the collection of sets K_t, $t_0 \leqslant t \leqslant \vartheta$, from /13/ (under the condition that $K_{t_0} \neq \phi$; however, if $K_{t_0} = \phi$, then Problem 2.1 does not have a solution /13/). The construction of sets K_t in the general case is very difficult. However, as in /1, 15/, we can delineate the cases when the sets K_t admit of an effective description. Let us discuss this question.

4. Everywhere below it is assumed that M is a convex closed bounded set in H_1. By the symbol N_{t_*} we denote the collection of all elements $x \in H_1$ with the property:

for every program $v(t),\ t_* \leqslant t \leqslant \vartheta$ there exists a program $u\ (t),\ \ t_* \leqslant t \leqslant \vartheta$, for which

$$x_{u,v}(\vartheta; t_*, x) = F(\vartheta - t_*)x + \int_{t_*}^{\vartheta} F(\vartheta - t)(A_2u(t) - A_3v(t) + w(t))\,dt \in M \tag{4.1}$$

Let $\{u\}_q$ denote the collection of all programs $u\,(t)$ on interval q. Using the theorem on the separability of convex sets /6, 17/, it is not difficult to verify (see/15/, for example) the validity of

Lemma 4.1. $x \in N_t$ if and only if

$$\gamma\ (t,\ x) = \sup_\eta \varphi\ (\eta,\ t,\ x) \leqslant 0, \quad \|\eta\|_1 \leqslant 1 \tag{4.2}$$

Here

$$\varphi(\eta, t, x) = \rho_v(\eta, t) - \rho_u(\eta, t) + \min_{q \in M} \langle \eta, q\rangle_1 -$$

$$\int_t^{\vartheta} \langle \eta, F(\vartheta - \xi)\, w(\xi)\rangle_1\, d\xi - \langle \eta, F(\vartheta - t)\, x\rangle_1$$

$$\rho_v(\eta, t) = \max_{v \in \{v\}_{[t,\vartheta]}} \int_t^{\vartheta} \langle \eta, F(\vartheta - \xi)\, A_3v(\xi)\rangle_1\, d\xi$$

$$\rho_u(\eta, t) = \max_{u \in \{u\}_{[t,\vartheta]}} \int_t^{\vartheta} \langle \eta, F(\vartheta - \xi)\, A_2u(\xi)\rangle_1\, d\xi$$

We introduce the following conditions.

Condition A. For any $t \in [t_0,\ \vartheta]$ the functional

$$\chi(\eta, t) = \rho_v(\eta, t) - \rho_u(\eta, t) + \min_{q \in M} \langle \eta, q\rangle_1 \tag{4.3}$$

is weakly upper-semicontinuous in H_1 and when $\gamma\ (t,\ x) > 0$ the upper bound in (4.2) is reached on the single element $\eta^\circ = \eta^\circ\ (t,\ x)$.

Condition B. For any $t \in (t_0,\ \vartheta)$ the set

$$G_u(t) = \left\{ g\,(u) = \int_{t_0}^{t} F\,(t - \xi)\, A_2u\,(\xi)\, d\xi \;\middle|\; u \in \{u\}_{[t_0,t]} \right\} \tag{4.4}$$

is compact in H_1.

Note 4.1. Condition A corresponds here to the regularity condition from the theory of position differential games /1, 2/. This condition is fulfilled, for example, if for any $t \in [t_0, \vartheta)$ the functional $\chi\,(\eta,\ t)$ of (4.3) is concave in η (see Corollary 2 in /13/). As a matter of fact, in this case /18/ the functional mentioned is weakly upper-semicontinuous in H_1 for each $t \in [t_0, \vartheta)$ and, further, when $\gamma\,(t,\, x) > 0$, by virtue of the strict convexity of a sphere in H_1, the upper bound of the positively homogeneous and concave functional $\varphi\,(\eta,\ t,\ x)$ is achieved on a single element of the unit sphere. In its own turn, the concavity of $\chi\,(\eta,\ t)$ for any t holds, for example, if for any t a convex set $R\,(t) \subset H_1$ exists such that $F\,(\vartheta - t)\, A_2P = F\,(\vartheta - t)A_3Q + R\,(t)$. This is an analog of the uniformity condition from the theory of differential games /1, 2/. (An algebraic sum of sets stands on the right in (4.4)).

Note 4.2. Condition B is fulfilled, for example, for a wide class of parabolic systems (see /5/ and Sect. 5 below). It is fulfilled as well when $H_2 = E_n$ and $A_2u = bu$, where $b \in H_1$, since now the operator $g\,(u)$ from (4.4), mapping $L_2\,([t_0,\ t],\ E_n)$ into

H_1, is completely continuous (see /4/), as is easily verified.

Theorem 4.1. Let Conditions A and B be fulfilled. If $N_{t_0} \neq \phi$, then $N_t \neq \phi$, for any $t \in [t_0, \vartheta]$ and the system of sets N_t, $t_0 \leqslant t \leqslant \vartheta$ is strongly u-stable.

This assertion is proved by the plan of proof of Theorem 2.1 in /15/ and relies on the fixed point theorem in /19/ and on Lemmas 4.2, 4.3 following below.

Lemma 4.2. Under Condition A the element $\eta^\circ(t, x)$ is weakly continuous in x on the set $H_1 \setminus N_t$ for each fixed $t \in [t_0, \vartheta)$ in the following sense: if $\{x_k\} \subset H_1 \setminus N_t$, $x_k \to x \in H_1 \setminus N_t$, then $\eta^\circ(t, x_k) \rightharpoonup \eta^\circ(t, x)$.

The validity of Lemma 4.2 is directly verifiable.

Corollary. Under Condition A the element $\eta^\circ(t, x)$ is continuous on the set $H_1 \setminus N_t$ for each fixed $t \in [t_0, \vartheta]$.

This assertion follows immediately from Lemma 4.2 since $\|\eta^\circ(t, x)\| = 1$ for any $x \in H_1 \setminus N_t$ and space H_1 is reflexive.

Let us consider the following auxiliary problem.

Problem 4.1. Let $\gamma(t, x) > 0$ for some $t \in [t_0, \vartheta]$ and $x \in H_1$. Find the program $v_0(\xi)$, $t \leqslant \xi \leqslant \vartheta$ satisfying the condition: for every program $u(\xi)$, $t \leqslant \xi \leqslant \vartheta$

$$\min_{q \in M} \| x_{u, v_0}(\vartheta; t, x) - q \|_1 \geqslant \gamma(t, x)$$

Let $E(t, x)$ be the set of all elements $\eta^\circ(t, x)$ of the unit sphere in H_1, for which the upper bound in (4.2) is reached.

Lemma 4.3. Let the functional $\chi(\eta, t)$ of (4.3) be weakly upper-semicontinuous on H_1. If the program $v_0(\xi)$, $t \leqslant \xi \leqslant \vartheta$ solves Problem 4.1, then an element $\eta^\circ(t, x) \in E(t, x)$ exists for which

$$\int_t^\vartheta \langle \eta^\circ(t, x), F(\vartheta - \xi) A_3 v_0(\xi) \rangle_1 \, d\xi = \tag{4.5}$$

$$\max_{v \in \{v\}_{[t, \vartheta]}} \int_t^\vartheta \langle \eta^\circ(t, x), F(\vartheta - \xi) A_3 v(\xi) \rangle_1 \, d\xi$$

Conversely, every function satisfying the maximum condition (4.5) for some $\eta^\circ(t, x) \in E(t, x)$ is a solution of Problem 4.1.

The lemma's proof is based on the minimax theorem /4/.

By definition, $N_\vartheta = M$. From Theorems 3.1, 4.1, Theorem 1 in /13/, and Lemma 4.1 follows

Theorem 4.2. Let Conditions A and B be fulfilled. Problem 2.1 has a solution if and only if $\gamma(t_0, x_0) \leqslant 0$. When this inequality is fulfilled the problem is solved by the strategy (4.6)

$$U^e = U^e(t, x) = \{u^e \mid \langle A_2^*(y - x), u^e \rangle_2 = \max_{u \in P} \langle A_2^*(y - x), u \rangle_2\}$$

where now y is a (unique) element in N_t, closest to x.

Note 4.3. We assumed above that $\| F(t) \|_1 \leqslant$ exp ωt, $t \geqslant 0$, $\omega = \text{const}$, where $c \leqslant 1$ (this condition played an essential role in the proof of Lemma 3.1). In the general case of a strongly continuous semigroup the original Problem 2.1 can be imbedded in an analogous control problem in a wider Hilbert space in which there already holds the bound needed for the semigroup. As such a space we can take the completion H_0 of space H_1 with respect to the norm

$$\| x \|_0^2 = \int_0^\infty \| F(t) x \|_1^2 \exp(-\omega_1 t)\, dt, \quad \omega_1 > 2\omega$$

Let $F_0(t)$ be the closure of $F(t)$ from H_1 onto H_0. The semigroup $\{F_0(t), t \geqslant 0\}$ is strongly continuous and $\| F_0(t) \|_0 \leqslant \exp \omega_1 t$, $t \geqslant 0$ (for example, see /11/). Let us now examine Problem 2.1 in space H_0, choosing as the target set the closure M_0 of set M in the new norm. We call this new problem Problem 2.1°. If set M is bounded and weakly closed in H_1 (the latter is ensured, for example, by the convexity and closedness of M in H_1) and the sets $G_u(\vartheta)$, $G_v(\vartheta)$ (see Condition B) are compact in H_1, then it can be verified that such an imbedding is invariant in the sense that the strategy U solving Problem 2.1° (with $x_0 \in H_1$) also solves Problem 2.1 in space H_1.

5. The class of controlled systems being considered covers, in particular, certain controlled systems described by parabolic and hyperbolic partial differential equations. We present here an example (also see /5/) which is simple but very important in applications. Let Ω be a bounded region in the n-dimensional space E_n, with a $2k$ times continuously differentiable boundary Γ, situated locally to one side of Γ. Let $A(y, D) = \Sigma_{|\alpha| \leqslant 2k} a_\alpha(y) D^\alpha$ be a real differential expression, elliptic in $\bar\Omega = \Omega \bigcup \Gamma$, whose coefficients $a_\alpha(y)$ in $\bar\Omega$ are continuous and have continuous derivatives up to order $|\alpha|$, inclusive; $a_0(y) \geqslant \beta > 0$ where the number β is sufficiently large. We are given the controlled parabolic system

$$\frac{\partial z(t, y)}{\partial t} = -A(y, D) z(t, y) + u(t, y) - v(t, y) + w(t, y) \tag{5.1}$$

$$y \in \Omega, \quad t_0 < t \leqslant \vartheta, \quad z(t_0, y) = z_0 \in W_2^{2k}$$
$$z|_\Gamma = \partial z/\partial \nu|_\Gamma = \ldots = \partial^{k-1} z/\partial \nu^{k-1}|_\Gamma = 0, \quad t_0 < t \leqslant \vartheta \tag{5.2}$$

Here u, v, w are functions, measurable on $T = [t_0, \vartheta] \times \Omega$, belonging to space $L_2(T)$; for almost all t the controls $u \in P \subset L_2(\Omega)$, $v \in Q \subset L_2(\Omega)$, where P, Q are convex bounded closed sets; ν is the normal to Γ, outward relative to Ω; W_2^{2k} is the Sobolev space: $W_2^{2k} = \{z(y) \mid D^\alpha z \in L_2(\Omega), \ |\alpha| \leqslant 2k\}$. The operator $A_1 z(y) = -A(y, D) z(y)$, defined on the elements of W_2^{2k}, satisfying conditions (5.2) in the sense of imbedding theorems, generates an analytic contractive semigroup $\{F(t), t \geqslant 0\}$ in space $L_2(\Omega)$ (for example, see /5, 6, 8–11, 20/).

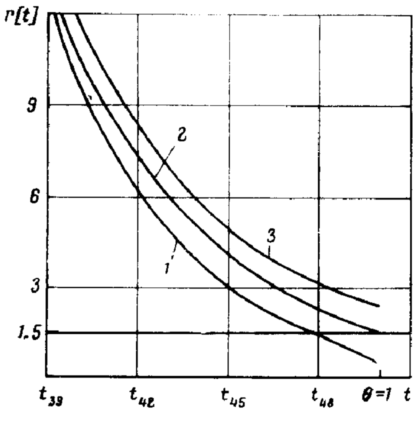

Fig. 1

Thus, we arrive at system (1.1) by setting $H_1 = H_2 = H_3 = L_2(\Omega)$, $x(t) \equiv z(t, \cdot)$, $x_0 = z_0$, $u(t) \equiv u(t, \cdot)$, $v(t) \equiv v(t, \cdot)$, $w(t) \equiv w(t, \cdot)$. The functions (2.1) are called the motions of system (5.1) corresponding to strategy U. Condition B is fulfilled here since the set G_u belongs to the domain of A_1^α by virtue of the estimate $\| A_1^\alpha F(t) \|_1 \leqslant c \cdot t^{-\alpha}$, $0 < \alpha < 1$, and of the boundedness of P, and, further, the operator $A_1^{-\alpha}$ is completely continuous in H_1 /8, 10, 20/.

In particular, let

$$\Omega = (0,\ 1),\quad t_0 = 0,\ \vartheta = 1,\ z\,(0,\ t) = z\,(1,\ t) = 0,\ z\,(0,\ y) = z_0\,(y)$$

Equation (5.1) has the form

$$\frac{\partial z\,(t, y)}{\partial t} = \frac{\partial^2 z\,(t, y)}{\partial y^2} + u\,(t, y) - v\,(t, y)$$

The sets $P = Q$ and M are closed spheres in $L_2\,(\Omega)$ of radii μ and λ. The encounter Problem 2.1 for Eq. (5.3) can be treated as the problem of heating a rod by distributed sources $u\,(t,\ y)$ under the condition of undetermined thermal interference $v\,(t,\ y)$. We can make use of Theorem 4.2 to construct the desired control function u. Condition A (see Note 4.1) and Condition B (see Sect. 5 above) are here fulfilled. The functional γ from (4.2) has the form

$$\gamma\,(t, x) = \Big\{\int_0^1 \Big[\int_0^1 G\,(y, \xi, \vartheta - t)\, x\,(\xi)\, d\xi\Big]^2 dy\Big\}^{1/2} - \lambda$$

$$G\,(y, \xi, t) = 2 \sum_{j=1}^{\infty} \exp\,(-(\pi j)^2 t) \sin j\pi y \sin j\pi\xi$$

where $G\,(y,\ \xi,\ t)$ is the function of the influence of the instantaneous point source. Let $\gamma\,(t_0,\ z_0) \leqslant 0$. The control u^e, dictated by the extremal strategy (4.6), solves the problem posed and is determined for almost all t by the rule: if $\gamma\,(t,\ z) \leqslant 0$, then u^e is any function from P; if $\gamma\,(t,\ z) > 0$, then

$$u^e = \mu\,(h_t\,(y) - z\,(y))\Big(\int_0^1 (h_t\,(y) - z\,(y))^2\, dy\Big)^{-1/2}$$

where the function $h_t\,(y)$ is the solution of the problem

$$\int_0^1 (h_t\,(y) - z\,(y))^2\, dy = \min_{h\,:\,\gamma(t,h)\leqslant 0} \int_0^1 (h\,(y) - z\,(y))^2\, dy$$

The motions

$$x\,(t)_\Delta = z\,(t, y)_\Delta = \int_0^1 G\,(y, \xi, t)\, z_0\,(\xi)\ d_\xi + \int_0^t\int_0^1 G\,(y, \xi, t - \tau)(u\,(\tau, \xi) - v\,(\tau, \xi))\, d\xi\, d\tau$$

were simulated on BESM-6 for the initial condition $z_0 = 1000\,y,\ 0 \leqslant y \leqslant 0.5$; $z_0 = 1000\,(1 - y),\ 0.5 \leqslant y \leqslant 1$ for the numbers $\lambda = 1.484$, $\mu = 10$, and for a partitioning Δ of the time interval $[0,\ 1]$ into equal parts of length $\delta = 0.02$. Figure 1 shows the curves

$$r\,(t) = \Big(\int_0^1 z^2\,(t,\ y)_\Delta\ dy\Big)^{1/2}$$

Curve *1* corresponds to the controls $u\,(t,\ y) = u^e\,(t,\ y,\ z)$, $v \equiv 0$; curve *2*, to the controls $u\,(t,\ y) = u^e\,(t,\ y,\ z)$, $v\,(t,\ y) = u^e\,(t,\ y,\ z)$; curve *3*, to the controls $u\,(t,\ y) \equiv \mu$, $v\,(t,\ y) \equiv 0$.

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Translated by N. H. C.